The Effect of Wiggler Imperfections on Nonlinear Harmonic Generation in Free-Electron Lasers

Henry P. Freund, Sandra G. Biedron, *Member, IEEE*, Stephen V. Milton, and Heinz-Dieter Nuhn

***Abstract*—The generation of harmonics through a nonlinear mechanism driven by bunching at the fundamental has sparked interest in using this process as a path toward an X-ray free-electron laser (FEL). An important issue in this regard is the sensitivity of the nonlinear harmonic generation to wiggler imperfections. Typically, linear instabilities in FELs are characterized by increasing sensitivity to both electron beam and wiggler quality with increasing harmonic number. However, since the nonlinear harmonic generation mechanism is driven by the growth of the fundamental, the sensitivity of the nonlinear harmonic mechanism is not severely greater than that of the fundamental. In this paper, we study the effects of wiggler imperfections on the nonlinear harmonics in a 1.5-Å FEL, and show that the decline in the third harmonic emission with increasing levels of wiggler imperfections roughly tracks that of the fundamental.**

***Index Terms*—Electromagnetic radiation, electron beam applications, free-electron lasers, frequency conversion, frequency-domain analysis, nonlinearities, nonlinear differential equations, simulation.**

I. INTRODUCTION

THE GENERATION of harmonics through a nonlinear mechanism driven by bunching at the fundamental, which has been studied in 1-D [1], [2] and 3-D analyses [3]–[5], has sparked interest in using this process as a path toward an X-ray free-electron laser (FEL). Studies are in progress for an FEL to serve as a next-generation light source. The current design for the Linac Coherent Light Source (LCLS) at SLAC [6] employs an electron beam with an energy of 14.35 GeV and a peak current of 3400 A in conjunction with a planar wiggler with an amplitude of 13.2 kG and a period of 3.0 cm (yielding a wiggler strength parameter K =3.7) to generate 1.5-Å X-rays using the fundamental interaction. However, at this short wavelength the interaction is extremely sensitive to wiggler imperfections. In this paper, we study the effect of wiggler imperfections on the nonlinear harmonic generation mechanism. Typically, linear instabilities in FELs show an increasing sensitivity to both electron beam and wiggler quality as the harmonic number increases. However, in the nonlinear mechanism, the harmonics are driven by the growth of the fundamental, and it may be expected, therefore, that the sensitivity of the harmonics to wiggler imperfections mirrors that of the fundamental. As we will show in this paper, that is, indeed, the case.

The specific example considered is the above-mentioned LCLS. The nominal emittance and energy spread for the electron beam are derived from radio frequency (RF) photocathode gun and linac simulations for a single-wavelength "slice" of the beam that indicate a normalized emittance $\varepsilon_n = 1.0\pi$–1.5π mm-mrad and an energy spread of 0.006% can be achieved. In addition, the LCLS wiggler uses multiple segments (with an amplitude of 13.2 kG and a period of 3.0 cm) of a flat-pole face design with strong focusing in the gaps. The resonant wavelength is near 1.5 Å. For simplicity, we assume a single-segment parabolic pole face (PPF) wiggler, thereby eliminating the necessity of any external focusing. In addition, we assume the beam is characterized by a slice energy spread and emittance of 0.006% and 1.1π mm-mrad, respectively.

The 3-D nonlinear, polychromatic simulation code MEDUSA [3], [5], [7]–[10] is used to simulate the nonlinear generation of harmonics. MEDUSA uses planar wiggler geometry and treats the electromagnetic field as a superposition of Gauss–Hermite modes. A Gaussian electron beam distribution is used in energy and phase space. The field equations are integrated simultaneously with the 3-D Lorentz force equations for an ensemble of electrons. No wiggler averaging is imposed on the orbit equations, and MEDUSA is capable of propagating the electron beam through arbitrary magnetic structures and self-consistently treating the associated evolution of the electromagnetic field.

It should be remarked that linear harmonic instabilities coexist with the nonlinear generation process. However, since the growth rates of the linear harmonic instabilities decrease with increasing harmonic number, and since these linear harmonic instabilities are more sensitive to beam quality than is the nonlinear generation mechanism, the linear harmonic instabilities are typically overwhelmed by the nonlinear process. Nevertheless, both harmonic mechanisms are implicitly included in the simulation.

For simplicity, we consider a PPF wiggler for enhanced focusing [11], [12]. The natural focusing in this wiggler model is weaker than that in the actual LCLS design; hence, both the betatron period and gain length will be longer than the design values. However, the PPF wiggler will serve as a convenient wiggler model to demonstrate the sensitivity of the nonlinear

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harmonic mechanism to wiggler imperfections. The PPF wiggler can be represented as

$$\begin{aligned}\mathbf{B}_w(\mathbf{x}) = {}& [B_w(z) + \Delta B_w(z)] \\ & \cdot \left\{ \cos k_w z \left[\hat{\mathbf{e}}_x \sinh\left(\frac{k_w x}{\sqrt{2}}\right) \sinh\left(\frac{k_w y}{\sqrt{2}}\right) \right.\right. \\ & \left. + \ \hat{\mathbf{e}}_y \cosh\left(\frac{k_w x}{\sqrt{2}}\right) \cosh\left(\frac{k_w y}{\sqrt{2}}\right) \right] \\ & \left. -\sqrt{2}\,\hat{\mathbf{e}}_z \cosh\left(\frac{k_w x}{\sqrt{2}}\right) \sinh\left(\frac{k_w y}{\sqrt{2}}\right) \sin k_w z \right\} \quad (1)\end{aligned}$$

where k_w denotes the wiggler wavenumber for a wiggler period λ_w, $B_w(z)$, and $\Delta B_w(z)$ denote the systematic (i.e., nonrandom) and random variations in the amplitude, respectively. The systematic variation in the wiggler amplitude is assumed to be

$$B_w(z) = \begin{cases} B_w \sin^2\left(\dfrac{k_w z}{4N_w}\right), & 0 \le z \le N_w\lambda_w \\ B_w, & N_w\lambda_w < z \end{cases} \quad (2)$$

which describes an adiabatic entry taper over the first N_w wiggler periods.

The random component of the amplitude is chosen at regular intervals using a random number generator, and a continuous map is used between these points. Since a particular wiggler may have several sets of pole faces per wiggler period, the interval is chosen to be $\Delta z = \lambda_w/N_p$, where N_p is the number of pole faces per wiggler period. Hence, a random sequence of amplitudes $\{\Delta B_n\}$ is generated, where $\Delta B_n \equiv \Delta B_w(n\Delta z)$. The only restriction is that $\Delta B_w = 0$ over the entry taper region [i.e., $\Delta B_n = 0$ for $0 \le n \le 1+N_pN_w$] to ensure a positive amplitude. The variation in $\Delta B_w(z)$ between these points is given by

$$\Delta B_w(n\Delta z + \delta z) = \Delta B_n + [\Delta B_{n+1} - \Delta B_n]\sin^2\left(\frac{\pi}{2}\frac{\delta z}{\Delta z}\right) \quad (3)$$

where $0 \le \delta z \le \Delta z$. In the rest of this paper, it shall be assumed, for simplicity, that $N_p = 2$. Observe that it is possible to model the effects of pole-to-pole variations in specific wiggler magnets with this formulation.

It is useful at the outset to characterize the performance for an ideal wiggler (i.e., $\Delta B_w = 0$). Then we plot the growth of the fundamental and the third harmonic in Fig. 1 under the assumptions that the initial power in the fundamental corresponds to the spontaneous noise level of 482 W [13] while the third harmonic is undriven. The fundamental saturates at a power level of 13.66 GW over a length of 119 m with a gain length of approximately 5.30 m. This is within about 9.7% of the prediction from the linear analytic theory of 4.83 m [13]. The third harmonic reaches a maximum power level of 275 MW over a length of about 120 m. As pointed out earlier [1], [3]–[5], the gain length for the nonlinear harmonic generation mechanism scales inversely with the harmonic number. We find a harmonic gain length of 1.73 m in the simulation, which is in close accordance with this scaling law.

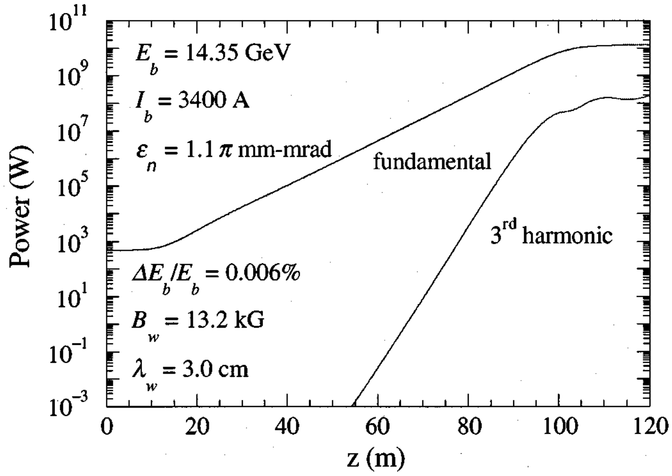

Fig. 1. Evolution of the power in the fundamental and third harmonic for an ideal wiggler.

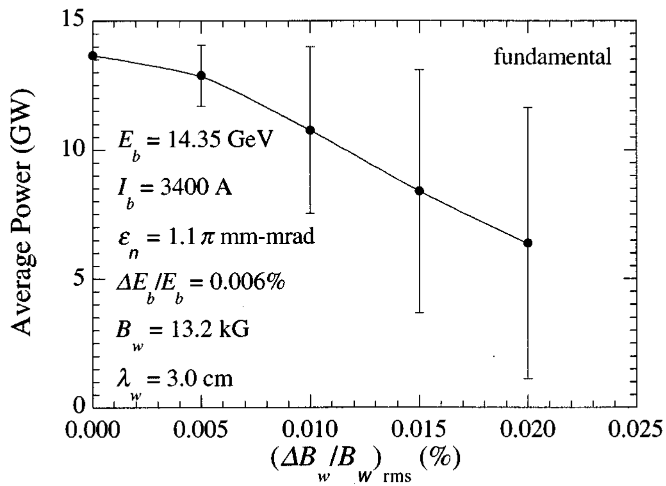

Fig. 2. Variation in the saturated power at the fundamental with the rms level of wiggler imperfections.

In studying the effect of wiggler imperfections, we choose an rms fluctuation level and generate multiple fluctuation distributions using a random number generator. Using a sufficiently large number of such distributions, we can determine the fluctuation statistics to within an arbitrary degree of accuracy. In practice, we find that using 30 different randomly generated fluctuation distributions yields an accuracy of within about 1% for the ensemble average of the saturated power. Note that it makes little sense to discuss the effect of wiggler imperfections on the gain length because, in many cases, the fluctuations in the beam trajectories due to these imperfections result in no clear region of exponential growth.

The decline in the average power level in the fundamental with increases in the rms fluctuation level of the wiggler imperfections is shown in Fig. 2, where the error bars denote the standard deviation. Note that the average power decreases by almost a factor of two as $(\Delta B_w/B_w)_{\rm rms}$ increases to 0.02%. In

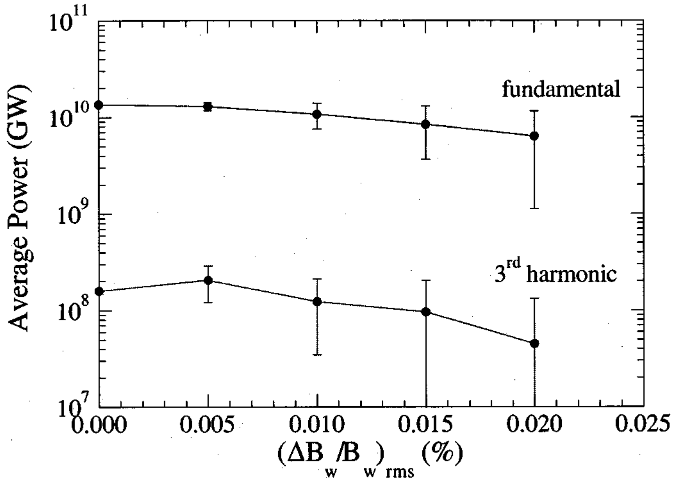

Fig. 3. Variation in the saturated power at the fundamental and third harmonic with the rms level of wiggler imperfections.

addition, the standard deviation increases with increasing levels of wiggler imperfections. It is important to recognize that these results are statistical in nature. With careful sorting of individual magnets, an actual wiggler can be expected to produce performance anywhere within the ranges shown in the error bars. Hence, for the optimal magnet sorting, there need be no performance penalty unless the variations about the mean in the magnetizations of the individual magnets comprising the wiggler exceed about 0.015%.

Now that the effects of wiggler imperfections on the fundamental have been characterized, we proceed to the effects on the third harmonic. To this end, we plot the variation in the saturated power at both the fundamental and third harmonic in Fig. 3 on a logarithmic scale. It is clear from the figure that the variation in the average saturation power levels is not as regular as that at the fundamental; however, the general decrease in power levels as $(\Delta B_w/B_w)_{\rm rms}$ increases to 0.02% is comparable for the fundamental and the third harmonic. Indeed, the ratio of the power in the third harmonic to that in the fundamental remains relatively constant at about 1.2% (within the error bars) across the entire range of wiggler imperfections studied.

In summary, a three-dimensional, polychromatic simulation code has been employed in studying the effect of wiggler imperfections on nonlinear harmonic generation in high-gain FELs, including those in the X-ray regime. It is well known that the linear harmonic instability in FELs is more sensitive to both electron beam and wiggler quality than is the fundamental. However, since the nonlinear harmonic mechanism relies on the growth of the fundamental and the corresponding bunching of the electron beam, it was expected that the effect of wiggler imperfections on the nonlinear harmonic mechanism would track the effect on the fundamental. This expectation was, indeed, borne out in simulation. For parameters consistent with the LCLS design, the power ratio in the saturated power in the third harmonic to that in the fundamental remained relatively constant, to within the error bars, as the rms fluctuation level of the wiggler imperfections rose to 0.02%.

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